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Hyperbolic Paraboloid Slab Foundation

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Abstract: *Traditionally, slab foundation is made up of reinforced flat plates. Replacing these plates by others with a hyperbolic paraboloid geometry can be considered an alternative for structural foundations of residential, commercial, or industrial buildings, especially in soils with low bearing capacity. When compared to traditional slab foundations, the hyperbolic paraboloid foundation reduces the consumption of materials (steel, wood, and concrete), which helps to reduce the environmental impact caused by the extraction of raw materials. Although this type of foundation is easy to build, certain precautions must be taken into account during construction. Therefore, this article describes the practical characteristics related with each construction stage of the foundation, namely the precautions to be taken with the location, excavation, execution of the forms and waterproofing of the foundation modules*

Keywords: *Foundation, slab foundation, shell foundation, hyperbolic paraboloid, construction process;*

1. Introduction

Foundation engineering is a very old activity, as old as the art of building itself, and it can certainly be said that this activity is present in all buildings developed by man [1]. Since the 1930s, analytical theories of soil behaviour have been developed and perfected, including the work of Terzaghi and Fröhlich [2], Taylor [3] and Skempton and MacDonald [4]. The first recognised use of shell foundations dates to the 1950s, through the architect Felix Candela [5], who is the pioneer of shell foundations. Candela was concerned with style and elegance, and this was his incentive to explore the use of shells as a structural element. This motivation led to an extensive exploration of shell-shaped structural models, many of which can still be found today. One example is the hyperbolic paraboloids, commonly known as hypar shell footing, which were first used by renowned architect Felix Candela on Mexican soil. Later, the hyperbolic paraboloid foundation was developed and suited for high-rise buildings and used for elevated water tank structures in soil with low bearing capacity.

In Brazil, Dr Ronei Lombardi Filgueiras led the way in this art, where since the beginning of the 80s of the last century, where carrying out several projects using hyperbolic paraboloid slab foundations [6].

Shell foundations, due to their curved shape, have greater stiffness and strength to comparison than structural elements with a flat surface. The strength of shell foundations is mainly due to their shape, which means that a minimum of material can be used to obtain maximum structural benefits. While a flat foundation like a roof slab, to resist vertical loads including self-weight, arise internal forces of bending and shear, a shell foundation, such as hyperbolic paraboloid shells, supports the applied loads mainly through internal forces in your plan, mainly tensions compression and in-plane shear. A shell may balance an applied transverse loading at the expense of the membrane stresses mainly, with bending actions minimized [7]. This difference in structural behaviour leads to optimization in the structural design of shell foundations since the material is subject mainly to internal normal compression forces. The economy of material becomes more evident when such elements of foundation are made of structural concrete, whose high performance under action of internal forces of compression provides a great reduction in consumption of material [7].

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In the literature, several studies refer to the ability to reduce materials and improve geotechnical performance through the application of shell foundations, the first known modern example being that of Felix Candela, built in Mexico in 1953.

Martins et al. [8] performed a technical and economic analysis of the adoption of shell as a solution for the foundation of an industrial vertical silo, where a rigid and flat slab foundation was initially designed. The results showed that steel consumption was reduced by 23 % of the initial value, while structural concrete consumption was reduced to about 60 %.

According to Abdel-Rahman [9] the most significant cost saving in the use of shell foundations was a 72 % gain when compared to a conventional raft foundation and 55 % when compared to a pile foundation in a factory project in Konnagar, India.

Ribeiro et al. [10] mentioned that when several solutions are possible in a construction project, it is important to reflect on the various alternatives and choose the one that promotes the lowest cost, as well as the shortest execution time. Although the use of hyperbolic paraboloid slab foundations has advantages, particularly due to its geometric particularity, which leads to economical solutions in terms of the consumption of materials used in execution (steel and concrete), the extra costs of construction due to labour is a disincentive for its application and because of this its use has not been widespread.

2. Economic Feasibility of Shell Foundations

In order to meet the demands of the world's growing population and consequent rapid urbanisation, it is expected that the existing building area could double by 2060 [11]. This demand will require substantial investment in new buildings. According to Adam et al, due to typical design and construction practices, the construction sector is responsible for at least 11 % of global greenhouse gas emissions. This makes it imperative to consider buildings as a key component in a low-carbon future.

In a project, elements such as the structural frame and the foundations are often those that most contribute to embodied carbon and therefore represent the systems with the greatest potential for limiting emissions [11][12]. According to Pratt [13], material consumption in a building's foundations can represent up to 40 % of the total amount of embodied carbon spent on its construction.

The obvious economic advantages of using shell-shaped foundation structures have been proven in several reports of applications in various parts of the world [14].

Feitckert and Muelle [12], based on existing structural design methodologies, calculated the quantities of materials to assess the environmental impact of shell foundations compared to conventional flat foundations. They found that shell foundations represent significantly less embodied carbon to support the same applied load. The embodied carbon of a shell footing under an applied load of 5 MN is 72 % less than the embodied carbon of a flat footing under the same load. This percentage decreases as the applied load decreases, but the saving is still significant. For an applied load of 1 MN, the shell footing requires 55 % less embodied carbon when compared to the flat footing for the same load.

Although the design of shell foundations has advantages, such as a reduction in the consumption of materials, they have not been widely used because flat foundations are more economically viable, especially in contexts where labour costs are higher. The cost of skilled labour to construct these foundations has hindered their implementation [15] [16]. This justifies more widespread use in countries where high material-to-labour cost ratio is one of the typical characteristics of their economy. Modern digital fabrication techniques, such as 3D soil printing, can a more cost-effective way to obtaining more complex shapes, mitigating the need for additional labour or expense [12]. Projects such as Teshima Art Museum, built in 2010, in Teshima, Japan, have shown that soil can be used as formwork for concrete structures (Fig. 1 a). The use of soil as formwork has advantages that include the fact that it is highly mouldable, the material itself is inexpensive and it is reusable, reducing construction waste [17]. Soil offers an opportunity for low- cost, waste-free formwork.

The use of soil as formwork is not a new concept, particularly in the design and manufacture of shells. Heinz Isler coined the term "freely shaped hill" in his article 'New Shapes for Shells' (1960) as a method of designing shells, where soil is moulded into the desired shape and concrete is poured on top (Fig. 1 b). Felix Candela used this method to make in-situ shell foundations in 1953, in which soft clay was roughly cut into shape and then refined using a simple form, or template, which was then coated with mortar [5]. These examples require skilled labourers to manually prepare the soil to act as formwork. This process is time-consuming and costly, which may explain why shell foundations have not been widely used and why their use is conditional on an analysis of labour costs. However, innovation in digital tools and advanced manufacturing has led to customised manufacture of complex geometries, while reducing the material and labour required to build these systems.

a) b)

Fig. 1 a) Earth formwork for the Teshima Art, b) Isler's freely shaped hill method [12]

3. Case Study

This section describes the foundation construction of a residential condominium in the state of Minas Gerais, Brazil, using the hyperbolic paraboloid slab foundation model, thus pointing out the practical characteristics related to the construction stages and the precautions that were taken when laying the foundations.

The Fig. 2 shows the hyperbolic paraboloid slab foundation cross section. Soil foundation was classified in accordance with Unified Soil Classification System (USCS), as MH sandy elastic silt

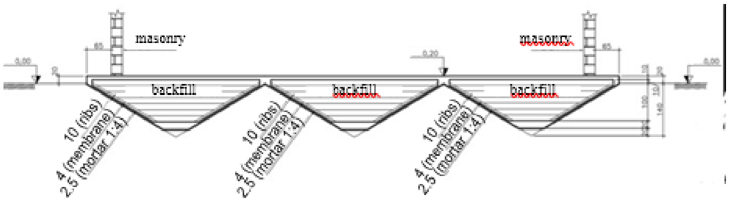

Fig. 2 Hyperbolic paraboloid slab foundation cross section

3.1. Location of the Hyperbolic Paraboloid Membrane Modules

The construction process begins with locating the modules, marking the edges of the building according to the project and then, with the help of topography, marking the dimensions of each module (Fig. 3). The modules are then aligned, and the dimensions of each module are placed on the land using stakes.

Fig. 3 Localisation of the modules with the help of topography

3.2. Excavation of the Membrane Modules

The modules are excavated using mechanical equipment. The excavation is no more than 1.4 m deep and is carried out from the edge of each module to the center of the convergence of the membrane (Fig. 4).

Fig. 4 Excavation of the membrane modules

Once the modules are in the desired shape (pyramidal), the modules are adjusted and smoothed. This operation is carried out manually using a template made of steel rods, which guides the direction of the excavation of the curving guidelines of each module (Fig. 5). The soil on the surface of the modules is then moisturized and compacted using manual tools.

Fig. 5 Regularisation of modules using the template

3.3. Laying the First Layer of Mortar

The regularization of the modules is completed by laying a layer of cement/sand mortar with a 1:4 ratio, with an average thickness of 2.5 cm. This process is carried out using a template, as shown in Fig. 6. The day after laying the mortar layer, the template should be removed from the module and then these points should be leveled with mortar, as shown in Fig. 7. The purpose of laying this layer of mortar is not only to regularize the modules, but also to waterproof and protect the steel reinforcements from contact with the ground

Fig. 6 Applying the mortar layer for regularization

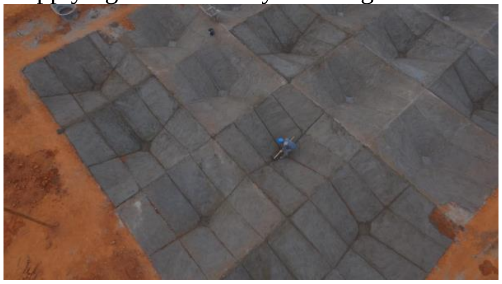

Fig. 7 Removal of module templates

3.4. Building Installations

Once the modules have been leveled, the primary and secondary sewage and rainwater installations are laid, as shown in Fig. 8. These installations are carried out in accordance with the project and are laid on clean, level ground. The pipes are then covered with plastic sheeting to protect the reinforcements from contact with the ground

Fig. 8 Building Installations

3.5. Shape of the Modules on the Foundation Perimeter

Fig. 9 shows the placement of formwork panels installed along the external perimeter of the building. The purpose of these panels is to safeguard the shape of the molds located around the foundation and to allow concreting of the rib placed at the top edge of each module. The joints of the panels must be well fixed so that there are no gaps. The forms must be locked in place using inclined struts, tie rods, tensioners, wedges, etc., according to the size of the panels and the placement load to be borne. At this stage, the panels should also be checked for lateral and vertical alignment, level, and watertightness.

Fig. 9 Shape of the modules on the perimeter of the foundation

3.6. Membrane Module Reinforcement

In each membrane module, reinforcement is placed in the ribs located at the top edges of the module and in the planes. The rib reinforcements with stirrups are identical in all modules.

Steel reinforcement with 18 cm square mesh and 6.3 mm diameter CA-50 steel are placed in the membrane planes. In the middle of the membrane planes, 5 CA-50 steel bars with a diameter of 12.5 mm were distributed in order to connect the ribs at the opposite ends of each membrane. These membrane reinforcements were installed in both directions of each module (Fig. 10).

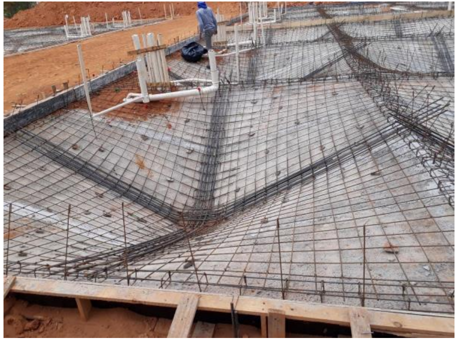

Fig. 10 Placing the steel reinforcement in the modules

3.7. Concreting the Modules

Type C25 structural concrete was used for concreting the modules. The membranes and ribs were concreted from the top to the bottom of the modules with a thickness of 4 cm. Fig. 11 illustrates the concreting of the modules.

Fig. 11 Concreting the modules

3.8. Concrete Curing

The concrete was cured immediately after concreting. The concrete cured can be realized by hydraulic curing, keeping the concrete moist by applying water, or by chemical curing. Fig. 12 shows the application of chemical curing after concreting the modules.

Fig. 12 Chemical curing after concreting the modules

3.9. Backfill the Modules

After the concrete is cured, the modules are backfilled. The soil removed from the excavation of the modules (described in B) was reused to backfill the modules, using 20 cm thick layers of backfill, mechanically compacted to the level of the ribs of each module, as shown in Fig. 13. The reuse of soil from the excavation to backfill the modules provides a clean site that is easy to move around and avoids the environmental impact associated with transporting and depositing waste in a dump.

Fig. 13 Backfill the modules

3.10. Second Layer of Regularising Mortar on the Embankment

Once all the molds have been filled, the embankment is waterproofed by applying a layer of mortar on top of the compacted soil (see Fig. 14). This layer of cement/sand mortar with a 1:4 ratio and a thickness of 2.5 cm is identical to the one applied to regularize the planes of the module membranes (described in C).

Fig. 14 Applying of the mortar layer to regularise the embankment

3.11. Slab Steel Reinforcement and Building Installations

After the mortar layer, the positive reinforcement of the compression table is positioned, consisting of a 15 cm square EQ- 61 mesh in CA- 60 steel (Fig. 15). As shown in Fig. 15, the electrical and telecommunications installations are also placed, and the negative reinforcement is then assembled.

Fig. 15 Assembly steel reinforcement of slab and building installations

3.12. Slab Concreting

Once all the activities have been completed, the concreting of the slab begins, with a thickness of 10 cm. A vibrating ruler used during the work, making circular movements to better thicken and level the concrete, avoiding the formation of voids (Fig. 16). In the three days following concreting, the concrete is cured by spraying it with water once a day, as shown in Fig. 17. Once the concrete is cured, the foundation consisting of the hyperbolic paraboloid slab is ready to receive the loads from the superstructure.

Fig. 16 Slab concreting

Fig. 17 Concrete curing after slab concreting

4. Conclusion

With building construction expected to continue at a high rate to meet demands of rapid urbanisation because of a growing world population and the necessity of mitigate climate change, it is imperative that the construction sector considers alternative building processes to traditional ones.

Shell foundations are presented as an alternative to conventional flat foundations due to their material-saving potential. In this work, the constructive steps of foundation of a residential condominium in the state of Minas Gerais, Brazil, using the hyperbolic paraboloid slab foundation model were described. Off this description, we considered the relevance of the following:

1) During the excavation process, the use of a template with the shape of the modules facilitates the process of cutting and adjusting the planes that form the membrane;
2) The layers of mortar on the membrane planes makes it easier to tie down the reinforcements and protect them from direct contact with the ground;
3) The modules are concreted quickly and easily due to the low consumption of concrete, given the reduced thickness of the membrane (around 4 cm);
4) Easy passage of electrical, telecommunications and water pipes, etc.
5) Wood consumption is very low, required only for the outer perimeter of the foundation;
6) Reusing the excavated soil to fill in the modules makes for a clean and tidy construction site and avoids the economic and environmental burdens of transporting and depositing soil;
7) The slab foundation makes it easier to start building the structure, as it makes it possible to mark out the location of the walls
8) The screed is practically ready once the slab foundation iscompleted
9) The sidewalks around the building are already taken into account in the slab foundation
10) The low consumption of materials needed to build the foundation makes this method economically and environmentally advantageous, as it helps to reduce the environmental impact associated with the extraction of raw materials such as aggregates, steel, and wood

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