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Propagation of Electromagnetic Waves in an Anisotropic Monoclinic Medium

Almas Kurmanov, Nurlybek Ispulov, Almar Zhumabekov and Kairat Dossumbekov

Toraighyrov University, Pavlodar 140008, Kazakhstan

Abstract: *The article uses the analytical matrizant method to study electromagnetic waves in anisotropic monoclinic dielectric media. In the low-frequency approximation for homogeneous infinite media, one of the variants of obtaining the indicatrix equations is presented. For the same class of media, the solution of Maxwell's equations in the form of an averaged matricant is obtained.*

Keywords: *matricant, anisotropic medium, monoclinic*

1. Initial ratios

Consider dielectric monocrystals of monoclinic symmetry with a symmetry axis of the 2nd order $A_2 \parallel Oz$. The dielectric constant tensor for them has the form [1]:

$$\hat{\varepsilon} = \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & 0 \\ \varepsilon_{xy} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}.$$

By the matrizant method, Maxwell 's equations are reduced to a matrix of coefficients , which in this case:

$$\hat{B} = \begin{bmatrix} 0 & b_{12} & b_{13} & 0 \\ b_{21} & 0 & 0 & b_{24} \\ -b_{24} & 0 & 0 & b_{34} \\ 0 & -b_{13} & b_{43} & 0 \end{bmatrix}, \qquad \frac{d}{dz}\begin{pmatrix} E_y \\ H_x \\ H_y \\ E_x \end{pmatrix} = \hat{B}\begin{pmatrix} E_y \\ H_x \\ H_y \\ E_x \end{pmatrix} \tag{1}$$

Where

$$b_{12} = i(\omega\mu_0\mu - \frac{k^2{}_y}{\omega\varepsilon_0\varepsilon_z});\; b_{13} = i\frac{k_x k_y}{\omega\varepsilon_0\varepsilon_z}; \qquad b_{21} = i[\omega\varepsilon_0\varepsilon_y - \frac{k^2{}_x}{\omega\mu_0\mu}]; \tag{2}$$

$$b_{24} = i[\omega\varepsilon_0\varepsilon_{xy} + \frac{k_x k_y}{\omega\mu_0\mu}]; \qquad b_{34} = -i(\omega\varepsilon_0\varepsilon_x - \frac{k_y^2}{\omega\mu_0\mu}); \qquad b_{43} = i(\frac{k_x^2}{\omega\varepsilon_0\varepsilon_z} - \omega\mu_0\mu).$$

It is assumed that the vectors of the strengths and inductions of the electromagnetic field are harmonically dependent on time, i.e.:

$$\vec{E}, \vec{H}, \vec{B}, \vec{D}(\omega, \vec{r}) = \vec{E}, \vec{H}, \vec{B}, \vec{D}(\omega, z)\, e^{i\omega t}$$

The matricant is the normalized solution of equation (1), represented as an exponential series:

$$T = E + \int_0^z B dz_1 + \int_0^z \int_0^{z_1} B(z_1)B(z_2)dz_1 dz_2 + \ldots$$

$$T^{-1} = E - \int_0^z B dz_1 + \int_0^z \int_0^{z_1} B(z_2) B(z_1) dz_1 dz_2 - \dots$$

For a periodically inhomogeneous medium, in the presence of n periods, the matrix $\vec{u} = (E_y, H_x, H_y, E_x)^t$ it is represented in the form:

$$\vec{u}_n = T^n \vec{u}_0$$

where T – a matrix of one period of heterogeneity (the so-called monodromy matrix),

$\vec{u}_0$ and $\vec{u}_n$ – column matrices (1) for zero and periods of heterogeneity n.

Introduction of the matrix:

$$P = \frac{1}{2}(T + T^{-1})$$

gives a recurrence relation:

$$T^2 = 2PT - E$$

the consistent application of which makes it possible to obtain a dependence in the form of Chebyshev matrix polynomials of the second kind [1]:

$$T^n = 2\ P_n(P)\ T - P_{n-1}(P)\, E \qquad (3)$$

Consider the physico-mechanical averaged parameters of periodically inhomogeneous media.

The initial relations are the dispersion equations

$$\cos \tilde{k}_i h = \tilde{p}_i\,, \qquad (4)$$

The averaging of the medium will be carried out under the condition $\lambda >> h$ (where λ – wavelength, h– period of heterogeneity). How $\tilde{k}h = \frac{2\pi h}{\lambda} << 1$ we obtain a decomposition of the dispersion equations in the form:

$$\cos \tilde{k}_i h \cong 1 - \frac{\tilde{k}_i^2 h^2}{2} \qquad \text{or} \qquad \sqrt{1 - \tilde{p}_i^2} \approx \tilde{k}_i h = \frac{\tilde{k}_i H}{n}\,, \qquad (5)$$

where $H = nh$ – total layer thickness, n– the number of periods in the layer.

In formulas (5) $\tilde{p}_i$ are the roots of the characteristic equation following from the condition:

$$\det(P_{(2)} - \lambda E) = 0\,, \qquad (6)$$

$$P_{(2)} = E + \frac{< B >^2\ h^2}{2}\,, \qquad (7)$$

where $< B > = \frac{1}{h}\int_0^h B dz$.

Assuming in (5) $\tilde{p}_i \approx 1$, we also have:

$$T - \tilde{p}_i E \cong \langle B \rangle h\,.$$

Under such conditions, the matrix (3) for the averaged medium can be written as:

$$\langle T \rangle = \sum_{i=1}^{2} P_i [\hat{E} \cos \tilde{k}_i z + \frac{< B >}{\tilde{k}_i} \sin \tilde{k}_i z] \qquad (8)$$

where $P_i = \frac{P - \tilde{p}_i\, E}{\tilde{p}_i - \tilde{p}_j}$, *(i, j=1,2, i≠j)*

Let's show the fairness of equality:

$$P_n(\tilde{p}_i)T - P_{n-1}(\tilde{p}_i)E = E\cos\tilde{k}_i z + \frac{<B>}{\tilde{k}_i}\sin\tilde{k}_i z\,.$$

$$P_n(\tilde{p}_i)T - P_{n-1}(\tilde{p}_i)E = \frac{1}{2i\sqrt{1-\tilde{p}_i^{\,2}}}\{[(\tilde{p}_i + i\sqrt{1-\tilde{p}_i^{\,2}})^n - (\tilde{p}_i - i\sqrt{1-\tilde{p}_i^{\,2}})^n]T$$

$$-[(\tilde{p}_i + i\sqrt{1-\tilde{p}_i^{\,2}})^n(\tilde{p}_i - i\sqrt{1-\tilde{p}_i^{\,2}}) - (\tilde{p}_i - i\sqrt{1-\tilde{p}_i^{\,2}})^n(\tilde{p}_i + i\sqrt{1-\tilde{p}_i^{\,2}})]E\} =$$

$$= \frac{1}{2i\sqrt{1-\tilde{p}_i^{\,2}}}[(\tilde{p}_i + i\sqrt{1-\tilde{p}_i^{\,2}})^n - (\tilde{p}_i - i\sqrt{1-\tilde{p}_i^{\,2}})^n](T-\tilde{p}_iE) + \frac{1}{2}[(\tilde{p}_i + i\sqrt{1-\tilde{p}_i^{\,2}})^n + (\tilde{p}_i - i\sqrt{1-\tilde{p}_i^{\,2}})^n]E =$$

$$= \frac{1}{2i\tilde{k}_i h}[(1+i\frac{\tilde{k}_iH}{n})^n - (1-i\frac{\tilde{k}_iH}{n})^n]\langle B\rangle h + \frac{1}{2}[(1+i\frac{\tilde{k}_iH}{n})^n + (1-i\frac{\tilde{k}_iH}{n})^n]E$$

Assuming for the z coordinates, the values of which, due to averaging, significantly exceed the period of inhomogeneity h, that is, when $n \to \infty$ $(1 \pm i\frac{kH}{n})^n = e^{\pm ikz}$, we get:

$$P_n(\tilde{p}_i)T - P_{n-1}(\tilde{p}_i)E = E\cos\tilde{k}_i z + \frac{<B>}{\tilde{k}_i}\sin\tilde{k}_i z\,.$$

Along with the construction of the matrix (8), knowledge of the roots makes it possible to obtain the equations of the indicatrix of electromagnetic waves of different polarization. The indicatrix equations [2] for homogeneous anisotropic media follow from the low-frequency decomposition of the dispersion equations (4). Then, deciding $\langle B\rangle = B_0$ from (4) and (5) we get:

$$\tilde{k}_i^{\,2} = \frac{2(1-\tilde{p}_{i(2)})}{h^2}\,. \qquad (9)$$

2. Calculations

According to (1) and (7) we obtain the matrix $P_{(2)}$:

$$P_{(2)} = \begin{pmatrix} p_{11} & 0 & 0 & p_{14} \\ 0 & p_{11} & -p_{14} & 0 \\ 0 & -p_{14} & p_{33} & 0 \\ p_{14} & 0 & 0 & p_{33} \end{pmatrix} =$$

$$= \begin{pmatrix} 1+\frac{1}{2}(b_{12}b_{21}-b_{13}b_{24})h^2 & 0 & 0 & \frac{1}{2}(b_{12}b_{24}+b_{13}b_{34})h^2 \\ 0 & 1+\frac{1}{2}(b_{12}b_{21}-b_{13}b_{24})h^2 & 0 & 0 \\ 0 & -\frac{1}{2}(b_{12}b_{24}+b_{13}b_{34})h^2 & 1+\frac{1}{2}(b_{34}b_{43}-b_{13}b_{24})h^2 & 0 \\ -\frac{1}{2}(b_{13}b_{21}+b_{43}b_{24})h^2 & 0 & 0 & 1+\frac{1}{2}(b_{34}b_{43}-b_{13}b_{24})h^2 \end{pmatrix}$$

Solve (6) to find the values $\tilde{p}_i$:

$$\tilde{p}_{1,2} = 1 + \frac{1}{4}h^2(b_{12}b_{21} - 2b_{13}b_{24} + b_{34}b_{43} \mp$$

$$\mp \frac{h^2}{4}\sqrt{\left(b_{12}^2b_{21}^2 - 4b_{12}b_{13}b_{21}b_{24} - 4b_{13}^2b_{21}b_{34} - 4b_{21}b_{24}^2b_{43} - 2b_{12}b_{21}b_{34}b_{43} - 4b_{13}b_{24}b_{34}b_{43} + b_{34}^2b_{43}^2\right)}$$

According to (9) we obtain the equations of the indicatrix:

$$k_{1,2}^2 = \frac{1}{2}(-b_{12}b_{21} + 2b_{13}b_{24} - b_{34}b_{43} \pm$$

$$\pm\sqrt{\left(b_{12}^2 b_{21}^2 - 4b_{12}b_{13}b_{21}b_{24} - 4b_{13}^2 b_{21}b_{34} - 4b_{21}b_{24}^2 b_{43} - 2b_{12}b_{21}b_{34}b_{43} - 4b_{13}b_{24}b_{34}b_{43} + b_{34}^2 b_{43}^2\right)})$$

From the latter expressions, it is possible to obtain the values of the propagation velocities of TE and TM waves in homogeneous anisotropic media:

$$\vartheta_1 = \frac{\omega}{k_1}, \ \ \vartheta_2 = \frac{\omega}{k_2} \tag{10}$$

Let an electromagnetic wave with a frequency $\omega = 3*10^8$ Hz distributed in a crystal HIO_3 $\hat{\varepsilon} = \{7,2;8,0;6,9\}$, $\varepsilon_{xy} = 0$, $\mu = 1$ Hn/m, $\varepsilon_0 = 8{,}8510^{-12}$ F/m.

Then the formula (8) for the averaged matricant will take the form (fig.1, Wolfram Mathematica the program is used):

$$T = \begin{pmatrix} P1\,Cos[k1\,z] + P2\,Cos[k2\,z] & P1\frac{b12\,Sin[k1\,z]}{k1} + P2\frac{b12\,Sin[k2\,z]}{k2} & P1\frac{b13\,Sin[k1\,z]}{k1} + P2\frac{b13\,Sin[k2\,z]}{k2} & 0 \\ P1\frac{b21\,Sin[k1\,z]}{k1} + P2\frac{b21\,Sin[k2\,z]}{k2} & P1\,Cos[k1\,z] + P2\,Cos[k2\,z] & 0 & P1\frac{b24\,Sin[k1\,z]}{k1} + P2\frac{b24\,Sin[k2\,z]}{k2} \\ -P1\frac{b24\,Sin[k1\,z]}{k1} - P2\frac{b24\,Sin[k2\,z]}{k2} & 0 & P1\,Cos[k1\,z] + P2\,Cos[k2\,z] & P1\frac{b34\,Sin[k1\,z]}{k1} + P2\frac{b34\,Sin[k2\,z]}{k2} \\ 0 & -P1\frac{b13\,Sin[k1\,z]}{k1} - P2\frac{b13\,Sin[k2\,z]}{k2} & P1\frac{b43\,Sin[k1\,z]}{k1} + P2\frac{b43Sin[k2\,z]}{k2} & P1\,Cos[k1\,z] + P2\,Cos[k2\,z] \end{pmatrix}$$

$$P1 = \begin{pmatrix} \frac{2+b12\,b21\,h^2-b13\,b24\,h^2-2\,L1}{2\,L1-2\,L2} & 0 & 0 & \frac{(b12\,b24+b13\,b34)\,h^2}{2\,(L1-L2)} \\ 0 & \frac{2+b12\,b21\,h^2-b13\,b24\,h^2-2\,L1}{2\,L1-2\,L2} & \frac{(b13\,b21+b24\,b43)\,h^2}{2\,(L1-L2)} & 0 \\ 0 & -\frac{(b12\,b24+b13\,b34)\,h^2}{2\,(L1-L2)} & \frac{2-b13\,b24\,h^2+b34\,b43\,h^2-2\,L1}{2\,L1-2\,L2} & 0 \\ -\frac{(b13\,b21+b24\,b43)\,h^2}{2\,(L1-L2)} & 0 & 0 & \frac{2-b13\,b24\,h^2+b34\,b43\,h^2-2\,L1}{2\,L1-2\,L2} \end{pmatrix}$$

$$P2 = \begin{pmatrix} \frac{1+\frac{1}{2}(b12\,b21-b13\,b24)\,h^2-L2}{-L1+L2} & 0 & 0 & \frac{(b12\,b24+b13\,b34)\,h^2}{2\,(-L1+L2)} \\ 0 & \frac{1+\frac{1}{2}(b12\,b21-b13\,b24)\,h^2-L2}{-L1+L2} & \frac{(b13\,b21+b24\,b43)\,h^2}{2\,(-L1+L2)} & 0 \\ 0 & \frac{(b12\,b24+b13\,b34)\,h^2}{2\,(L1-L2)} & \frac{1+\frac{1}{2}(-b13\,b24+b34\,b43)\,h^2-L2}{-L1+L2} & 0 \\ \frac{(b13\,b21+b24\,b43)\,h^2}{2\,(L1-L2)} & 0 & 0 & \frac{1+\frac{1}{2}(-b13\,b24+b34\,b43)\,h^2-L2}{-L1+L2} \end{pmatrix}$$

$$L1 = \frac{1}{4}\left(4 + (b12\,b21 - 2\,b13\,b24 + b34\,b43)\,h^2 - \sqrt{\left(\left(b12^2\,b21^2 - 2\,b12\left(2\,b13\,b21\,b24 + 2\,b24^2\,b43 + b21\,b34\,b43\right) + b34\left(-4\,b13^2\,b21 - 4\,b13\,b24\,b43 + b34\,b43^2\right)\right)h^4\right)}\right);$$

$$L2 = \frac{1}{4}\left(4 + (b12\,b21 - 2\,b13\,b24 + b34\,b43)\,h^2 + \sqrt{\left(\left(b12^2\,b21^2 - 2\,b12\left(2\,b13\,b21\,b24 + 2\,b24^2\,b43 + b21\,b34\,b43\right) + b34\left(-4\,b13^2\,b21 - 4\,b13\,b24\,b43 + b34\,b43^2\right)\right)h^4\right)}\right)$$

,

Fig. 1 The formula of the average matricant

3. Conclusion

The problem of propagation of electromagnetic waves in anisotropic media is formulated and solved. The indicatrix equations and the averaged matricant for homogeneous media of monoclinic syngony are presented in an analytical form. The results obtained can be used in the design of devices, modeling and processing experiments of the interaction of electromagnetic waves with matter.

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